

Is this the longest Chess game?

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1 Introduction

In my experience, most chess games end in a few moves. If you want to play a lot of chess moves, you just play a lot of chess games. Still, there are games that seem to go on foreverrrrrrrrr. Perhaps the players are trying to *lull each other into a false sense of security while waiting for the moment to strike*, or perhaps they are stalling in a game of *Chess to the Death*.

Although many people “know how to play” chess, almost nobody fully understands the rules of chess, most authoritatively given by FIDE [1]. (See for example Figure 1 for a minor chess scandal that erupted in 2019 over an obscure corner case in the rules.) Several of these “deep cuts” have to do with game-ending conditions that were introduced to avoid interminable games.

Many chess moves are reversible (e.g. moving the knight forward and back to its starting position [4]), so informal games of chess could last forever with the players repeating a short cycle. In AD 1561, Ruy López added the “fifty-move rule” to prevent infinite games.¹ This rule (detailed below) ensures that irreversible moves are regularly played, and so the game always makes progress towards an end state. Another rule, “threefold repetition” also guarantees termination as a sort of backup plan (either of these rules would suffice on its own).

So, chess is formally a finite game. This is good for computer scientists, since it means that chess has a trivial $O(1)$ optimal solution. This allows us to move onto other important questions, like: What is the longest chess game? In this paper I show how to compute such a game, and then gratuitously present all of its 17,697 moves. Even if you are a chess expert (“chexpert”), I bet you will be surprised at some of the corner cases in the rules that are involved.

Speaking of rules, let’s first detail the three main rules that limit the length of the game. These rules cause the game to end in a draw (tie) when certain conditions are met.

1.1 The seventy-five move rule

The “fifty-move rule,” [8] as it is usually known, requires that an irreversible move is played at a minimum pace. For the sake of this rule, irreversible moves are considered captures and pawn moves,² including promotion and *en passant* (which is also a capturing move) but not the movement of a previously promoted piece. Specifically [1]:

9.3. The game is drawn, upon a correct claim by the player having the move, if:

- he writes his move on his scoresheet and declares to the arbiter his intention to make this move, which shall result in the last 50 moves having been made by each player without the movement of any pawn and without any capture, or
- the last 50 consecutive moves have been made by each player without the movement of any pawn and without any capture.

Note that what is provided here is the option for a player to *claim* a draw (and the two provisions essentially allow either player to claim the draw at the moment of the 50th move). If neither player is interested in a draw, either because they think their position is still winning, or are just trying to create the longest ever chess game, the game legally continues. That’s why what is actually relevant for this paper is provision 9.6, which defines a draw:

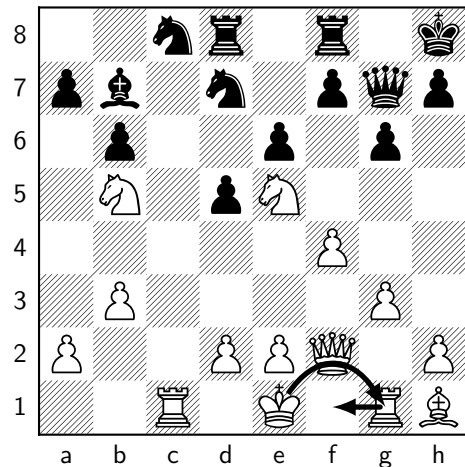


Figure 1: (*Nepomniachtchi – So, 2019.*) White to move during a speed Chess960 (aka “Fischer Random”) tournament. In this variant, the pieces start in different positions, but castling rules are such that the king and rook end up on the same squares that they would in normal chess. As a result, it is possible for the king or rook to not move during castling, or for the destination square for the king to already be occupied by the rook. Attempting to castle in the position depicted, grandmaster Ian Nepomniachtchi first touched the rook to move it out of the way. However, piece-touching rules require that when castling, the player first moves the king (and “Each move must be played with one hand only”); but how? The rook is occupying g1! One commenter suggested tossing the king into the air, then sliding the rook to f1 while the king is airborne, and then watching the king land dead center on its target. The arbiter required Nepo to make a rook move instead, but this was later appealed, and the game replayed.

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¹This rule only applied to games started after its introduction, so it is possible that some pre-1561 games are still in progress and may never end.
²Other irreversible moves include: Castling, moving a piece so as to lose castling rights, and declining to capture *en passant* (such capture must be made immediately, so if the option is not taken, it cannot be regained). The fifty-move rule could be soundly expanded to include these, but, that’s just like, not the rule, man.

9.6. If one or both of the following occur(s) then the game is drawn:

... 9.6.2. any series of at least 75 moves have been made by each player without the movement of any pawn and without any capture. If the last move resulted in check-mate, that shall take precedence.

Draws after 75 moves (per player, so really 150 moves) are compulsory.

Interestingly (at least as interesting as anything in this dubious affair) it is known that some otherwise winning endgame positions require more than 50 moves to execute (Figure 2). The rules of chess have at various times allowed for longer timers in such known situations, but were later simplified to the fixed 50 (and 75) move limit.

1.2 Fivefold repetition

The 75-move rule is rarely applied in practice, but its counterpart, “threefold repetition” [9] is often the cause of draws in modern chess. This rule states that if the same position appears three times, the players can claim a draw:

9.2.2. Positions are considered the same if and only if the same player has the move, pieces of the same kind and colour occupy the same squares and the possible moves of all the pieces of both players are the same. [...]

Like the 75-move rule, this rule has an optional version (upon three repetitions) and a mandatory one in 9.6:

[The game is a draw if ...]

9.6.1. the same position has appeared, as in 9.2.2, at least five times.

Fascinatingly (at least as fascinating as anything in this questionable undertaking), this rule used to require *consecutive* repetition of moves. However, there exist infinite sequences of moves with no consecutive n -fold repetition. For example, in the starting position, white and black can move either of their knights out and back. Let $\mathbf{0}$ be $\mathfrak{d}c3 \mathfrak{a}c6 \mathfrak{d}b1 \mathfrak{a}b8$ (queenside knights move, returning to the starting position) and $\mathbf{1}$ be $\mathfrak{d}f3 \mathfrak{a}f6 \mathfrak{d}g1 \mathfrak{a}g8$ (kingside). Now the Prouhet–Thue–Morse sequence [7] $0110100110010110\dots^3$ can be executed. This infinite sequence is cube-free (does not contain SSS for any non-empty string S) [6], and therefore never violates the consecutive threefold repetition rule [2].

Many implementations of chess ignore these rules or treat them incorrectly. Implementation of the seventy-five move rule simply requires a count of how many moves have transpired since a pawn move or capture, but programs typically do not force a draw after 75 moves. Repetition requires more work, since the program must keep track of each of the states reached since the last irreversible move. There are also some corner cases, such as ambiguity as to whether the starting position has “appeared” before the first move [4]. The ubiquitous FEN notation for describing chess positions does not even include any information about states previously reached.

³Let s_0 be the string “0”, then s_i is $s_{i-1}\overline{s_{i-1}}$ where $\overline{0} = 1$ and $\overline{1} = 0$.

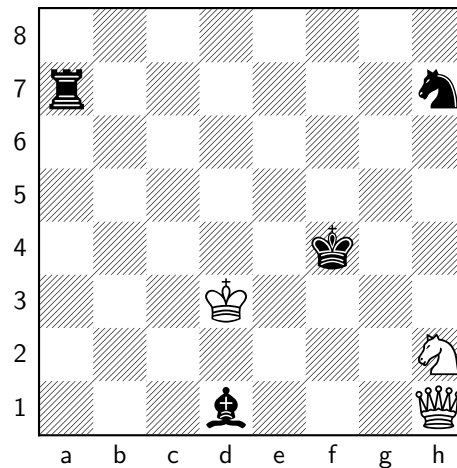


Figure 2: Black to move and mate in 545 moves (!). The position was found (by Zakharov and Makhnichev [10]) while building an endgame tablebase of all possible 7-piece positions. Of course, the game ends prematurely in a draw because of the 75-move rule.

1.3 Dead position

The informal version of this rule (“insufficient material”) states that if neither side has enough pieces to mate the opponent (for example, a king and bishop can never mate a bare king) then the game is drawn. Again, the formal rule is more subtle:

5.2.2. The game is drawn when a position has arisen in which neither player can checkmate the opponent’s king with any series of legal moves. The game is said to end in a ‘dead position’. This immediately ends the game...

This clearly includes the well-known material-based cases like king and knight vs. king, but it also surprisingly includes many other specific positions, especially those with forced captures (Figure 3).

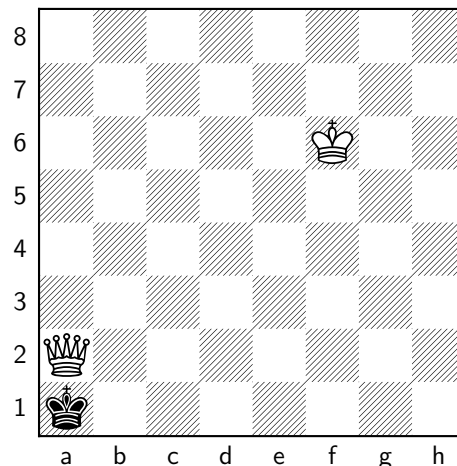


Figure 3: Black to move and draw in 0 (!). Most players and even chexperts believe that the only legal move is $Kxa2$, and that the game then ends in a draw with “insufficient material.” However, this game is *already over*. Since neither black nor white can win via any series of legal moves, by rule 5.2.2 the game immediately ends in a ‘dead position’. (Although see Section 1.3.1 for possible ambiguity in this rule.)

The insufficient material rule is curious in that it requires non-trivial computation to implement. In order to know whether the position is a draw, an implementation needs to be able to decide whether or not a series of moves that results in checkmate exists. Note that this is not nearly as bad as normal game tree search because the two sides can collaborate to produce the mate (it is not a $\exists\forall\exists\dots$ but rather $\exists\exists\exists\dots$). Still, such “helpmates” can still be quite deep (dozens of moves) and are interesting enough to be a common source of chess puzzles. Proving the non-existence of a helpmate can be very difficult indeed (Figure 4).⁴

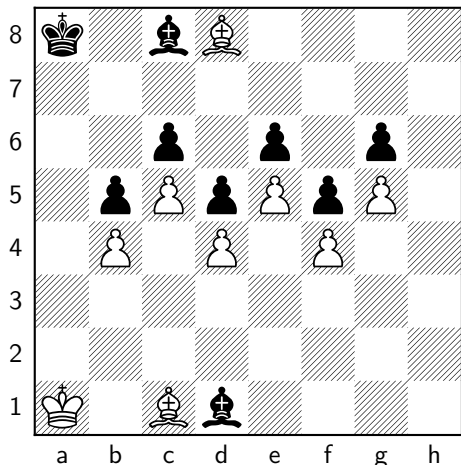


Figure 4: Thinking of implementing the rules of chess? To be correct, you’ll need your program to be able to deduce that no helpmates are possible in this position and thus the game is over. Stockfish rates this as +0.4 for white, even searching to depth 92. (Position is due to user `supercat` on Chess StackExchange.)

1.3.1 Ambiguity

Moreover, this rule contains some ambiguity. The phrase “any series of legal moves” is usually taken to mean something like, “the players alternate legal moves and follow most of the normal rules of chess.” In my opinion it is hard to justify an interpretation like this.

First, the rules specifically define “legal move”, with 3.10.1 saying “A move is *legal* when all the relevant requirements of Articles 3.1 – 3.9 have been fulfilled.” These requirements describe the movement of each piece as you are familiar (e.g. 3.3 “The rook may move to any square along the file or the rank on which it stands.”). They also disallow capturing one’s own pieces, or moving when in check. However, they allow as legal some moves that would otherwise be prohibited, like capturing the opponent’s king (this is excluded by 1.4.1, outside the definition of “legal”). Capturing the opponent’s king is generally not useful for demonstrating that checkmate is possible (Figure 5), so this is mostly a curiosity.

Second, what is a “series” of legal moves? It seems completely consistent to allow the white player to make several legal moves in a row, for example. The rules about alternating moves are again outside the definition of “legal move” and “series” is never defined.

We could instead interpret “any series of legal moves” as “taking the entirety of the rules of chess, any continuation of the game that ends in checkmate for either player.” I like this better, although it creates its own subtle issues. For example, should the position

⁴Perhaps an enterprising reader can prove that for generalized chess, it is NP-hard to decide whether the game has ended due to this rule?

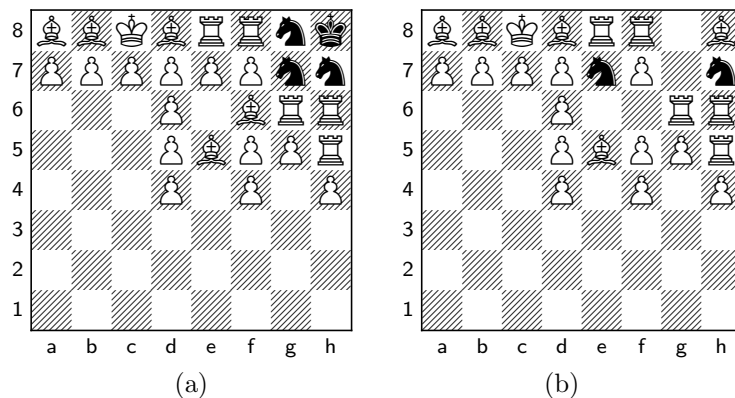


Figure 5: In this contrived and impossible position (a), white has many easy paths to mate. All of black’s pieces are pinned. There is also a “series of legal moves” where black mates white: ♜xg7++ ♝xh8?? ♞xe7++ . Capturing the king (by moving twice in a row) is a “legal move” (despite not being allowed by other rules), and doing so unpins black’s knight to deliver a smothered mate (b). Of course, abuse of this dubious technical possibility doesn’t change the status of the position, since we already have a mating sequence by white.

be considered dead if there is a checkmating sequence, but it requires entering a fivefold repetition or exhaustion of the 75-move rule? If so, this would end the game prematurely, and so it has implications for the longest possible game (Section 2.1). Even more esoterically, this interpretation causes the rule to be self-referential: A sequence must also be allowed by the rule being defined. A normal person would take the “least fixed point” (in the Kripke sense [3]) of this self-referential definition (fewest positions are drawn). But it is also consistent to interpret it *maximally*—in which case the longest chess game is zero moves!

For completeness, note that there are other routes to a draw (stalemate, draw offers) which we can ignore; it is easy to avoid these situations when generating the longest game.

2 Generating the longest game

It is generally not hard to avoid repeating positions, so the main obstacle we’ll face is the 75-move rule. Let’s call an irreversible move that resets the 75-move counter a *critical move*; this is a pawn move or capturing move (or both). The structure of the game will be a series of critical moves (I will call these “critical” moves) with a maximal sequence of pointless reversible moves in between them. If we execute the maximum number of critical moves and make 149 moves (just shy of triggering the compulsory version of the 75-move rule) between them, then this will be fairly easily seen as a maximal game.

In fact, for most positions, it is easy to waste 149 moves and return to the exact same position. So, the strategy for generating the longest game can mostly be broken into two tasks: Make a game with a maximum number of critical moves (it can also contain other moves) and then pad that game out to maximum-length excursions in between its critical moves.⁵

⁵The first phase was constructed by hand. Software for inserting excursions and checking the result is available at: sf.net/p/tom7misc/svn/HEAD/tree/trunk/chess/longest.cc

2.1 Maximal critical moves

Critical moves are pawn moves and captures. There are 16 pawns, each of which has 6 squares to move into before promoting, so this is 16×6 critical moves. There are 14 capturable pieces, plus the 16 pawns (they can be captured after promoting); capturing them nets an additional $16 + 14$ critical moves for a total of 126. Each critical move can be made after a maximum of $75 + 74$ reversible moves, giving $(75 + 74 + 1) \times 126 = 18900$ moves. The final move would capture the last piece, yielding a draw due to the remaining kings being insufficient material (“dead position”). This is our starting upper bound.

We will not quite be able to use the entire critical move budget. If pawns only move forward in single steps, they will eventually get blocked by the opposing pawn on the same file. Pawns can move diagonally off their starting file only by capturing. We have plenty of capturing to do anyway, so this is no problem. With four captures per side, the pawns can be doubled, with a clear route to promotion, like in Figure 6.

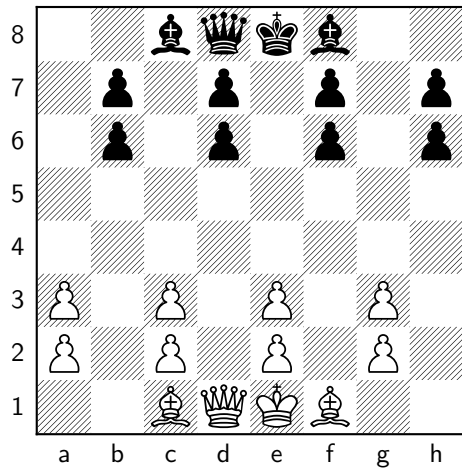


Figure 6: One way to clear the promotion routes for all pawns with eight captures.

However, each of these captures is *both* a pawn move *and* a capturing move. This means that we lose $4 + 4$ critical moves off our total budget. $150 \times (126 - 8) = 17700$ is the new upper bound.

Parity. If a critical move (such as a pawn move) is made by white, then the first non-critical move is made by black. The players alternate these pointless moves until black has made 75 and white 74. Now it is white’s move, and white must make a critical move or the game ends due to the 75-move rule. If instead we wanted black to make the next critical move, this would happen after black has made 74 and white 74 non-critical moves. Any time this happens we lose one move against the upper bound. So, we want to minimize the number of times we switch which player is making the critical moves. Obviously we must switch at least once, because both black and white must make critical moves. This reduces the upper bound to 17699.

Starting condition. The first critical move should be made by black. The starting position (with white to move) is analogous to the situation just described, as if black has just made a critical move, and it’s white’s turn. White will play 75 reversible moves,

black plays 74, and the 150th move is black making the first critical move.

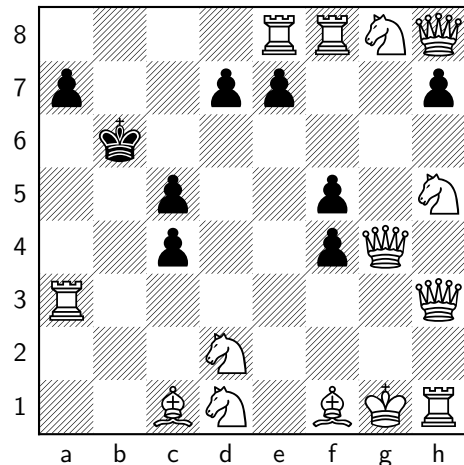
Note that white is quite constrained during this beginning phase, as pawn moves are critical moves and must be avoided. Only the white knights can escape the back rank. When we try to insert 149 pointless moves, we’ll only be able to move the knights and rooks, and doing this 75 times must leave e.g. one of the knights on an opposite-colored square.⁶ So we have to be a little careful about the position in which we make black’s first critical move.

Since white can only free their two knights, these are the only pieces that can be captured by black pawns. So it will not be possible for black to double four sets of their pawns as in Figure 6. This would require white to have a phase of critical moves to free pieces to capture, then black again to finish doubling pawns, then white again to promote its freed pawns. Each switch costs one move off the naive max. We can be more efficient with an asymmetric approach.

Black’s first phase of critical moves instead results in this:

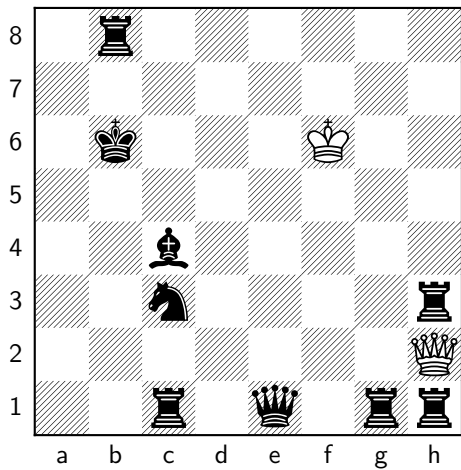


The white b and g pawns have a clear route to promotion. White can free each of the remaining 6 pawns with a single capture. Between this and black’s two pawn captures, this is the optimal 8 pawn moves that are also captures. Since black has plenty of freed pieces, white can promote all of their pawns during their own phase of critical moves, resulting in:

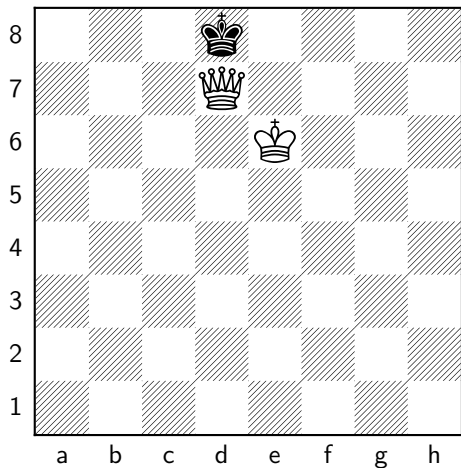


Now black can promote all of its pawns and capture white’s pieces. We actually leave the white queen; this turns out to be essential:

⁶Knight moves always change the color of the knight’s square, and same for rook moves constrained to the a1/b1 or g1/h1 squares.



Finally, white captures all of black's pieces, and mates the black king:



Since we switch which color is making critical moves a total of three times, we must come shy of the naïve maximum of 17,700 moves by three. This gives us an upper bound of 17,697 for this approach, which we will be able to achieve.

Ending condition. The way this game ends is subtle for several reasons. First, note that we left the white queen on the board and used it for mate. It is required that white be the one mating for parity reasons, similar to the reason black must make the first critical move. Since white makes critical moves in the last phase, black leads on non-critical moves; at the moment white makes the checkmating move, black has made 75 non-critical moves, and white 74. White's 75th move mates.

But doesn't this trigger the 75-move rule? No, this rule (9.6.2; Section 1.1) has a special exception for checkmate: "If the last move resulted in checkmate, that shall take precedence." Essentially, we can treat checkmate as a type of critical move.

Why not capture the white queen too? This would be a critical move, but once two kings remain, the game ends immediately in a draw. So there is no length advantage here over checkmate. In fact, attempting to capture may *shorten* the game: Consider the position in Figure 3 where black is *forced* to capture the queen. This game is over prior to the capture, so this is shorter than the checkmating sequence! Even if the king had an escape square, it is arguable (Section 1.3.1) that the game is over in any case: There is no "series of legal moves" that leads to mate. Either the king captures the queen (and then clearly no mate is possible with just the two kings) or the king escapes, but in doing so plays the 75th non-critical move, and triggers a draw by the 75-move rule.

The most foolproof way to ensure that mate is always possible is for the game itself to end in mate. This sidesteps any ambiguity

about the way the 75-move condition should be interpreted, as well.

2.2 Inserting excursions

The game described in Section 2.1 has 118 critical moves in 289 total moves. There is some inefficiency between the critical moves, but this doesn't matter since we are trying to generate a long game anyway. In fact, the next step will be to add *as much inefficiency as possible* in between the critical moves.

Each "critical section," which is the series of moves ending in a critical move, can be treated independently. If black ends the section with a critical move, then we want 75 + 74 non-critical moves to be played, and then black's critical move. For white, of course, 74 + 75. There are many ways we could try to make these 149 moves; we don't even have to use the moves that are already there as long as we end up in the right position to make the critical move. But a simple approach suffices.

For each critical section, we loop over all of the positions encountered, and attempt inserting excursions that return us to the same position but waste moves. There are two types of excursions we try: Even excursions (each player makes two moves) and odd (each player makes three moves).

Even excursions. This four move sequence moves two pieces of X_1 and X_2 of opposite colors. X_1 moves from s_1 to d_1 then X_2 from s_2 to d_2 , then X_1 moves from d_1 back to s_1 , and X_2 from d_2 back to s_2 . Easy. Any piece can perform this maneuver other than pawns (which would be critical moves anyway) as long as there are legal squares (considering check, etc.). All of s_1 , d_1 , s_2 , d_2 must be distinct. No shorter excursions are possible.

Odd excursions. This is the straightforward extension to three squares ($s_i \rightarrow m_i \rightarrow d_i \rightarrow s_i$), for a six-move sequence. The squares for each piece must be distinct but it is possible for e.g. m_2 to equal s_1 . Knights cannot perform this trick; each move changes the color of the square the knight sits on, which causes a contradiction with a cycle of length three. All other pieces can do it with sufficient room. The king, for example, can move horizontally, then diagonally, then vertically back to its starting square.

Note that odd excursions are not possible early in the game (prior to white moving any pawns), because even when the knights are free, the rooks only have two squares (and thus the same color parity argument applies as knights). Some opportunity can be created by having a black knight capture one of white's bishops. Fortunately, we do not need any odd excursions at this point in the game.

We find excursions by just looping over possible moves that satisfy the criteria, prioritizing odd excursions if the target (divided by two) is odd. In order to avoid triggering the fivefold repetition rule, we also keep track of all of the positions encountered, and never enter a position more than two times. (Here we avoid even threefold repetition.) It is not necessary to look beyond the critical section, because critical moves make it impossible to return to a prior position.

This process is not at all guaranteed to work; it may fail to fill the critical section. Indeed, as discussed in Section 2.1, we must have at least one move of slack whenever we switch from a critical move by one player to the other. In practice this approach succeeds readily, and manages to waste 149 moves in each critical section of the input game, save for the three times that parity requires one move of slack. The full game is uselessly included in a very tiny font in Section 4.

♀f2+ ♀c3 8393. ♀h4 ♀h3 8394. ♀g3 ♀hh1 8395. ♀h4 ♀g4 8396. ♀e1 ♀gg1 8397. ♀e6+ ♀a5 8398. ♀e1 ♀g5 8399. [♂xc1] ♀f3+ 8400. ♀e7 ♀fg3 8401. ♀d2 ♀f1 8402. ♀f2 ♀a4 8403. ♀f5 ♀g6 8404. ♀d5 ♀b5 8405. ♀a2+ ♀b4 8406. ♀c2 ♀a5 8407. ♀c1 ♀d1 8408. ♀g5 ♀d8 8409. ♀f6 ♀d6 8410. ♀e6 ♀gg8 8411. ♀e2+ ♀bg6 8412. ♀c2 ♀h6 8413. ♀e4 ♀a6 8414. ♀f6 ♀e8+ 8415. ♀f6 ♀b3 8416. ♀g7 ♀a8 8417. ♀g5 ♀d1 8418. ♀g6 ♀c8 8419. ♀g4 ♀a8 8420. ♀g6 ♀c3 8421. ♀g5 ♀g3 8422. ♀g4 ♀b3 8423. ♀g5 ♀hh8 8424. ♀g6 ♀h6 8425. ♀f6 ♀c8 8426. ♀g7 ♀f7 8427. ♀f6 ♀g8 8428. ♀e7 ♀e8+ 8429. ♀d7 ♀d8+ 8430. ♀e7 ♀f8 8431. ♀e4 ♀d8 8432. ♀c4+ ♀a5 8433. ♀e4 ♀h7 8434. ♀c2 ♀h6 8435. ♀d1 ♀hg6 8436. ♀c2 ♀b6 8437. ♀e4 ♀a5 8438. ♀g4 ♀gg8 8439. ♀e4 ♀e8 8440. ♀e6 ♀d6 8441. ♀g4 ♀g6 8442. ♀c6 ♀b3 8443. ♀f6 ♀bg3 8444. ♀h8 ♀b5 8445. ♀f6 ♀a7 8446. ♀g5+ ♀b5 8447. ♀c3 ♀d1 8448. ♀g5 ♀f6 8449. ♀c1 ♀fg6 8450. ♀c4 ♀f1 8451. ♀c1 ♀d4 8452. ♀c2 ♀b5 8453. ♀d1 ♀b4 8454. ♀c2 ♀gf3 8455. ♀a2 ♀fg3 8456. ♀h2 ♀a4 8457. ♀a2+ ♀a3 8458. ♀d5 ♀ag3 8459. ♀c6 ♀c3 8460. ♀d5 ♀c6 8461. ♀f5 ♀cg6 8462. ♀c5 ♀g5 8463. ♀f5 ♀h5 8464. ♀f2 ♀f7 8465. ♀f6 ♀a5 8466. ♀e7 ♀e8 8467. ♀d2 ♀f7 8468. ♀d6 ♀h1 8469. ♀e7 ♀h8 8470. ♀c1 ♀h1 8471. ♀f6 ♀f3+ 8472. ♀e7 ♀fh3 8473. ♀f6 ♀a6 8474. [♂xc3] ♀d5 8475. ♀b3 ♀c4 8476. ♀c2 ♀b3 8477. ♀g2 ♀b7 8478. ♀h2 ♀a5 8479. ♀b8 ♀f1+ 8480. ♀e7 ♀e5+ 8481. ♀d8 ♀b4 8482. ♀a7+ ♀b5 8483. ♀f2 ♀g1 8484. ♀d2 ♀c5 8485. ♀h6 ♀bb1 8486. ♀f6 ♀bc1 8487. ♀d6 ♀g3 8488. ♀e7 ♀g7 8489. ♀e3 ♀a5 8490. ♀e5 ♀c3 8491. ♀e3 ♀g3 8492. ♀e4 ♀a8 8493. ♀e7 ♀c3+ 8494. ♀e8 ♀d3 8495. ♀d8 ♀c6 8496. ♀e4 ♀a8 8497. ♀h7 ♀b7 8498. ♀e4 ♀g2 8499. ♀e3 ♀g3 8500. ♀c1 ♀g7 8501. ♀e3 ♀b5 8502. ♀e5 ♀a5 8503. ♀f5 ♀c1 8504. ♀e5 ♀e7 8505. ♀e3 ♀g7 8506. ♀c3+ ♀b5 8507. ♀c3 ♀a1 8508. ♀e7 ♀ac1 8509. ♀f8 ♀g3 8510. ♀e7 ♀cg1 8511. ♀d6 ♀gc1 8512. ♀f8 ♀gg1 8513. ♀d6 ♀b4 8514. ♀f6 ♀b5 8515. ♀f4 ♀b1 8516. ♀f6 ♀be1 8517. ♀h6 ♀b1 8518. ♀f8 ♀b4 8519. ♀h6 ♀a4 8520. ♀d2 ♀b5 8521. ♀f2 ♀e5 8522. ♀d2 ♀f1 8523. ♀f2 ♀a6 8524. ♀a7+ ♀b5 8525. ♀b8 ♀a6 8526. ♀d7 ♀b3 8527. ♀d8 ♀ef5 8528. ♀e7 ♀e5+ 8529. ♀d7 ♀a5 8530. ♀e7 ♀f3 8531. ♀f6 ♀b7+ 8532. ♀g6 ♀h1 8533. ♀f6 ♀bb1 8534. ♀h2 ♀b3 8535. ♀b2 ♀g5 8536. ♀h2 ♀c8 8537. ♀g2 ♀b7 8538. ♀f7 ♀e4 8539. ♀f6 ♀g4 8540. ♀c2 ♀g5 8541. ♀e7 ♀bh3 8542. ♀f6 ♀g4 8543. ♀b3 ♀g5 8544. ♀b1 ♀d5 8545. ♀b3 ♀g8 8546. ♀c3 ♀d5 8547. ♀d4 ♀f7 8548. ♀c3 ♀b6 8549. [♂xh3] ♀a7 8550. ♀e7 ♀a6 8551. ♀d6 ♀g6+ 8552. ♀e5 ♀d5 8553. ♀f1+ ♀b7 8554. ♀f5 ♀g7 8555. ♀c1 ♀g8 8556. ♀a1 ♀f8+ 8557. ♀e5 ♀a2 8558. ♀b1+ ♀b3 8559. ♀f1 ♀c2 8560. ♀f4 ♀c8 8561. ♀a4 ♀c7 8562. ♀f6 ♀h7 8563. ♀b3+ ♀c6 8564. ♀c2+ ♀d7 8565. ♀g7 ♀g8 8566. ♀c6+ ♀e7 8567. ♀c3 ♀c5 8568. ♀c2 ♀h7 8569. ♀c3 ♀g8 8570. ♀a5 ♀c7 8571. ♀c3 ♀d7 8572. ♀c6+ ♀e7 8573. ♀c2 ♀d7 8574. ♀h2 ♀h7 8575. ♀c2 ♀f5 8576. ♀f6 ♀h7 8577. ♀c2 ♀c6 8578. ♀c2+ ♀b6 8579. ♀b3+ ♀c6 8580. ♀d1 ♀b7 8581. ♀b3+ ♀a8 8582. ♀a4+ ♀b7 8583. ♀e6 ♀c2 8584. ♀f6 ♀h2 8585. ♀e5 ♀h1 8586. ♀e6 ♀c8 8587. ♀e5 ♀c3 8588. ♀f4 ♀c8 8589. ♀c3 ♀f8 8590. ♀f4 ♀fh8 8591. ♀f1 ♀f8 8592. ♀c4 ♀b3 8593. ♀f1 ♀c8 8594. ♀b1 ♀b7 8595. ♀d1 ♀a2 8596. ♀b1+ ♀c8 8597. ♀a1 ♀b7 8598. ♀d4 ♀d5 8599. ♀e5 ♀a8 8600. ♀f5 ♀f8+ 8601. ♀f6 ♀g8 8602. ♀a1 ♀a2 8603. ♀c1 ♀d5 8604. ♀c2 ♀g7 8605. ♀c1 ♀b3 8606. ♀f1 ♀d5 8607. ♀g1 ♀g6 8608. ♀f1 ♀gh6 8609. ♀e5 ♀g6 8610. ♀f2 ♀a6 8611. ♀f1+ ♀c4 8612. ♀h3 ♀d5 8613. ♀g4 ♀f7 8614. ♀h3 ♀g4 8615. ♀d6 ♀g6+ 8616. ♀e6 ♀g5 8617. ♀h3 ♀b1 8618. ♀e7 ♀h1 8619. ♀f3 ♀a7 8620. ♀h3 ♀g7 8621. ♀f6 ♀g5 8622. ♀a3+ ♀b6 8623. ♀h3 ♀c5 8624. [♂xf7] ♀gg1 8625. ♀f6 ♀h8 8626. ♀b3 ♀e1 8627. ♀a2 ♀e7+ 8628. ♀f6 ♀e3 8629. ♀g5 ♀ee8 8630. ♀a5+ ♀c4 8631. ♀g3 ♀c6+ 8632. ♀g5 ♀c6 8633. ♀e5 ♀g6+ 8634. ♀f5 ♀gh6 8635. ♀e2+ ♀b3 8636. ♀e4 ♀e8+ 8637. ♀d3 ♀ee6 8638. ♀f3 ♀e5 8639. ♀g2 ♀hh5 8640. ♀f3 ♀a3 8641. ♀h3 ♀h7 8642. ♀g2 ♀d7+ 8643. ♀c4 ♀de7 8644. ♀d3 ♀d7+ 8645. ♀c3 ♀h7 8646. ♀d3 ♀a7 8647. ♀h3 ♀h7 8648. ♀f5 ♀h5 8649. ♀h3 ♀b5 8650. ♀f3 ♀be5 8651. ♀f4 ♀b3 8652. ♀f3 ♀eg5 8653. ♀g2 ♀e5 8654. ♀c4 ♀h6 8655. ♀g2 ♀g5 8656. ♀f3 ♀e5 8657. ♀c6 ♀ce6 8658. ♀f3 ♀a4 8659. ♀c2 ♀b3 8660. ♀e4 ♀e8 8661. ♀c2 ♀d8+ 8662. ♀e4 ♀e8+ 8663. ♀f4 ♀eh8 8664. ♀e4 ♀g8 8665. ♀f5 ♀gh8 8666. ♀f2 ♀a4 8667. ♀e2+ ♀a5 8668. ♀e5+ ♀c4 8669. ♀d6 ♀g6 8670. ♀e5 ♀d6 8671. ♀g5 ♀g6+ 8672. ♀f4 ♀c6 8673. ♀g5 ♀d8 8674. ♀a5 ♀h8 8675. ♀f5 ♀e6 8676. ♀g5 ♀e5+ 8677. ♀f6 ♀e6+ 8678. ♀f5 ♀ee8 8679. ♀f6 ♀d4 8680. ♀g5 ♀c4 8681. ♀e1 ♀c5 8682. ♀a5+ ♀d4 8683. ♀a2 ♀c5 8684. ♀a6 ♀e3 8685. ♀a2 ♀c6 8686. ♀f6 ♀c5 8687. ♀f7 ♀e7+ 8688. ♀f6 ♀e1 8689. ♀f7 ♀d6 8690. ♀b3 ♀c5 8691. ♀a4 ♀g1 8692. ♀b3 ♀c1 8693. ♀g3 ♀g1 8694. ♀g6 ♀hh1 8695. ♀g3 ♀e1 8696. ♀h3 ♀eg1 8697. ♀h7 ♀g5 8698. ♀h3 ♀c6 8699. [♂xh1+] ♀b6 8700. ♀g1+ ♀a5 8701. ♀e1+ ♀a4 8702. ♀e8 ♀g1 8703. ♀a1+ ♀b4 8704. ♀e5 ♀b1 8705. ♀c5+ ♀b3 8706. ♀c7 ♀f1 8707. ♀a7 ♀g1 8708. ♀a1 ♀h1 8709. ♀g7 ♀b4 8710. ♀g2 ♀b3 8711. ♀g4 ♀h4 8712. ♀g2 ♀f4 8713. ♀c2+ ♀b4 8714. ♀c7 ♀f1 8715. ♀h2 ♀f7 8716. ♀h1 ♀c5 8717. ♀h6 ♀a7 8718. ♀h1 ♀f7 8719. ♀d5+ ♀b4 8720. ♀h1 ♀a5 8721. ♀h2 ♀b4 8722. ♀g3 ♀f1 8723. ♀h2 ♀b5 8724. ♀c7 ♀b4 8725. ♀d7 ♀f4 8726. ♀c7 ♀a4 8727. ♀c2+ ♀b4 8728. ♀d2+ ♀b3 8729. ♀c2+ ♀a3 8730. ♀g2 ♀b3 8731. ♀d7 ♀h4 8732. ♀e8 ♀c3 8733. ♀g4 ♀b3 8734. ♀d4 ♀h1 8735. ♀g4 ♀c2 8736. ♀g2+ ♀b3 8737. ♀c6 ♀b4 8738. ♀g2 ♀f1 8739. ♀g7 ♀h1 8740. ♀g5 ♀b3 8741. ♀g7 ♀h7 8742. ♀a1 ♀h1 8743. ♀f7 ♀g1 8744. ♀e8 ♀b1 8745. ♀a7 ♀g1 8746. ♀a8 ♀f1 8747. ♀a7 ♀f2 8748. ♀c7 ♀f1 8749. ♀e5 ♀b1 8750. ♀c7 ♀h1 8751. ♀c5 ♀b1 8752. ♀c1 ♀b4 8753. ♀c5+ ♀a4 8754. ♀e5 ♀b4 8755. ♀f8 ♀b1 8756. ♀e8 ♀a4 8757. ♀a1+ ♀b4 8758. ♀e1+ ♀a4 8759. ♀c3 ♀g5 8760. ♀e1 ♀h5 8761. ♀f7 ♀g5 8762. ♀d1+ ♀a5 8763. ♀e1+ ♀b5 8764. ♀g1 ♀a5 8765. ♀g2 ♀b6 8766. ♀g1+ ♀b5 8767. ♀h1 ♀b6 8768. ♀c1 ♀b5 8769. ♀a1 ♀c6 8770. ♀h1+ ♀d7 8771. ♀f6 ♀e8 8772. ♀g2 ♀g8 8773. ♀e6 ♀g7 8774. [♂xg7] ♀d8 8775. ♀h7 ♀c8 8776. ♀g7 ♀b8 8777. ♀f5 ♀a8 8778. ♀c3 ♀b7 8779. ♀g6 ♀a8 8780. ♀d3 ♀b8 8781. ♀f7 ♀c7 8782. ♀f6 ♀b7 8783. ♀d7+ ♀b8 8784. ♀d2 ♀c7 8785. ♀c3+ ♀b6 8786. ♀g6 ♀b7 8787. ♀c2 ♀a7 8788. ♀c7+ ♀a8 8789. ♀g3 ♀b7 8790. ♀g1 ♀c6 8791. ♀f1 ♀b7 8792. ♀g5 ♀a8 8793. ♀f4 ♀a7 8794. ♀c1 ♀a8 8795. ♀c3 ♀a7 8796. ♀c2 ♀a8 8797. ♀c8+ ♀a7 8798. ♀c7+ ♀a8 8799. ♀c8+ ♀a7 8800. ♀c2 ♀a8 8801. ♀c3 ♀a7 8802. ♀c1 ♀a8 8803. ♀f4 ♀a7 8804. ♀f1 ♀a8 8805. ♀d1 ♀b7 8806. ♀f1 ♀b8 8807. ♀g6 ♀b7 8808. ♀h3 ♀c6 8809. ♀f1 ♀c5 8810. ♀g1+ ♀c6 8811. ♀a1 ♀b7 8812. ♀g1 ♀a8 8813. ♀g3 ♀b7 8814. ♀c7+ ♀a8 8815. ♀d8+ ♀a7 8816. ♀c7+ ♀a6 8817. ♀c2 ♀a7 8818. ♀f2+ ♀b7 8819. ♀c2 ♀b6 8820. ♀c3 ♀b7 8821. ♀f6 ♀b6 8822. ♀b2+ ♀c7 8823. ♀c3+ ♀d6 8824. ♀d2+ ♀c7 8825. ♀c2+ ♀b8 8826. ♀d2 ♀a8 8827. ♀d7 ♀b8 8828. ♀d4 ♀b7 8829. ♀d7+ ♀a8 8830. ♀d3 ♀b7 8831. ♀b5+ ♀c7 8832. ♀d3 ♀b6 8833. ♀f7 ♀c7 8834. ♀h3 ♀b8 8835. ♀d3 ♀a7 8836. ♀g6 ♀b8 8837. ♀c3 ♀a8 8838. ♀d3 ♀b7 8839. ♀c3 ♀a8 8840. ♀f5 ♀b7 8841. ♀g7+ ♀a8 8842. ♀g6 ♀b8 8843. ♀g7 ♀c8 8844. ♀c6 ♀b8 8845. ♀h7 ♀c8 8846. ♀h6 ♀d8 8847. ♀h7 ♀e8 8848. ♀g7 ♀d8 8849. [♂d7+] 1-0

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