## COLOR- AND PIECE-BLIND CHESS

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## 1. Impressing humans

What better way for humans to impress each other with their brains, especially in movies, than to play chess-and to shout dramatically CHECKMATE! upon surprise-checkmating their opponent? Well, one way is to play chess while disadvantaged somehow, for example, by punching each other in the face repeatedly during the game to impair brain function (see Chess Boxing [8]). Another common distraction is to play a multitude of games against many opponents at the same time, in a so-called "simultaneous exhibition." The idea is that this is more challenging because of the need to maintain mental state for so many games at once, whereas your opponents only need to maintain state for one game. In truth, simultaneous exhibitions easily fall to a "man-in-the-middle attack." If the purported genius simply creates a perfect bipartite matching of the games played with the white pieces and the games played with black, he can mechanically forward moves between these pairs of boards. This requires only constant state (see next section) per pair of games, and guarantees an even score for the exhibition. So that's not very impressive.

Another disadvantage that humans sometimes use to impress each other is a blindfold (worn over the eyes). In this predicament they only hear the opponent announce moves and must imagine the position on the board in their mind's eye, both for the sake of remembering it and while exploring potential moves. Disadvantages can be combined, such as in the final scene of the 1988 documentary Bloodsport where Jean Claude van Damme is blinded by an illicit foreign substance during the final martial art battle. ${ }^{1}$

## 2. IMPRESSING COMPUTERS

In contrast, it is much more difficult to impress computers or impress people with computers. When it comes to computers playing chess, largely, the jig is up; it is now easy for chess programs, running on consumer hardware, to defeat the strongest human players. It is well known that striking a computer actually fixes it, so Chess Boxing becomes trivial. Blindfold chess is the natural interface for a chess computer; it is actually much more difficult to have the computer interpret the opponent's move by visually studying a physical board!

Playing multiple games simutaneously is an easy extension of playing a single game, although in principle the scale of

[^0]such a thing could still be impressive. This is also impressive to other computers, who are largely concerned with filling up their memories with efficiently coded data. With a modern chess engine, it is easy to scale to an arbitrary number of games, since the exhibitor can make progress by observing one of the boards, computing a strong move, and playing it; this requires $O(1)$ space because all of the state is stored externally in the exhibition itself. However, we run the risk of losing the tournament (other players may be yet stronger computers). The man-in-the-middle attack remains an efficient way to minimize loss (ensuring an exactly even score). The simplest way to do this is to explicily generate a perfect bipartite matching over the $n$ games $G$ being played. This consists of $n / 2$ pairs $\left\langle G_{w}, G_{b}\right\rangle$ (where we play as white against Bob and black against Walice, respectively). Since each game starts in the starting position, this is very easy; we can just assign the matches consecutively. Along with each pair we also record which of the following states we are in:
(1) We are waiting for a move from Walice (our white opponent)
(2) We have seen a move from Walice, which is $\qquad$ -.
(3) We are waiting for a move from Bob (our black opponent)
(4) We have seen a move from Bob, which is .

If in State 1, we just watch $G_{b}$ until Walice makes a move, then record it and proceed to State 2. We consume the move and move to State 3 by playing that move in $G_{w}$ against Bob (where it must be our turn). We can immediately seek out that game or wait until we naturally come upon it. However, we should only approach $G_{w}$ when the pair of games is in State 3, etc., otherwise we will not have a move to play.

There are $n / 2$ pairs, with two bits for the state, no more ${ }^{2}$ than $\log _{2}(64 \times 64 \times 4)=14$ bits for each move (source square, destination square, and 2 bits to distinguish promotion to queen, rook, bishop, or knight). However, we also need to store the matching of $G_{w}$ to $G_{b}$; this can be done with a pair of indices (or e.g. memory addresses) but unfortunately, this requires $\log _{2}(n)$ bits to represent. So overall this approach requires $O(n \log (n))$ space to play a simultaneous exhibition of $n$ games.

It appears to be possible to reduce the space usage per game to a constant. In order to perform a man-in-the-middle attack, we need a perfect matching between the white games and black games. It is not essential that the matching be stable over time; for example if we are forwarding moves between Walice and Bob, and between Waluigi and Bario, and these games happen to transpose to the same position, then it works just fine to switch to forwarding between Walice and Bario; Waluigi and Bob. So, rather than store the matching explicitly, we can reconstruct it from the stored state at each step.

Let's think about the process of forwarding the moves from our white opponents to our black opponents; the reverse is of course symmetric. The first step will be to wait for the

[^1]white opponents to make their moves, and then copy these $n / 2$ positions (not moves) into a vector in memory.

The black opponents are waiting for a move from us. Next, we'll copy their $n / 2$ positions into memory, aligned with the $n / 2$ positions already present. Let's say that the relation that defines a legal move in chess is

$$
B \xrightarrow{m} B^{\prime}
$$

where $B$ is the position before the move $m$, and $B^{\prime}$ is the resulting position. Our goal is to align the games such that $B^{\prime}$ (a position copied from our white opponent) is aligned with $B$ (a position pending a move for our black opponent) in memory; this will represent the perfect matching. Computing $m$ from $B$ and $B^{\prime}$ when $B \xrightarrow{m} B^{\prime}$ is easy (and unique), so this allows us to read off and play the move for each row.

By invariant, it will be possible to produce such an alignment. For example, the first time we do this, each $B$ will be the starting position, and $B^{\prime}$ will be a legal move made by white from the starting position. Any alignment will work. Let's say that just one of the white opponents played 1. d 4 , resulting in $B_{0}$; then one black opponent will make a legal response to this (say 1. ...Nf6, giving $B_{1}$ ). Then $B_{1}$ can be $B^{\prime}$ for the next round, which we can align with $B=B_{0}$, and so on.

The only tricky thing is figuring out which boards go with which. Although if $B \xrightarrow{m} B^{\prime}$ it is easy to deduce $m$, it is not possible to compute $B$ from $B^{\prime}$, or even from $B^{\prime}$ and $m$. This is because we may have both $B_{a} \xrightarrow{m_{a}} B^{\prime}$ and $B_{b} \xrightarrow{m_{b}} B^{\prime}$ with $B_{a} \neq B_{b}$. For example with $B^{\prime}$

$\ldots$ we could have $B_{a}$ and $B_{b}$ be


Both of which can precede $B^{\prime}$ (the move is even the same: Kxa1). So it is not enough to greedily assign edges in our perfect match; if we choose the edge $B_{b}$ to go with $B^{\prime}$, we might later find $B_{2}^{\prime}$ :

... and have no possible matching board, since it cannot legally follow $B_{a}$.

Fortunately, we know that there exists a perfect matching (assuming all players are playing legal moves) and we can tell if we found one (i.e., we didn't get stuck). So, one strategy that works is to choose randomly when there is some ambiguity, and start over from the beginning if we ever get stuck. In practice this will be pretty efficient, since convergent board states are unusual. We only need a single pseudorandom pool for the entire process, so it can be $O(n)$ bits; this seems easily enough to generate all possible permutations of $n / 2$ items. Even $2^{2 n}$ grows much faster than $n!$. If we don't like the random approach, I believe it is also possible to compute next_permutation in constant space; so we can just explicitly try all orderings for $B^{\prime}$ (this takes exponential time).

Once we have paired up each $B$ and $B^{\prime}$, we simply compute the move (which we now know exists) and play it in that game. We then wait for the black opponents to play their moves, copy the resulting board states into our vector and repeat the above process (but flipping "black" and "white").

Although this is more involved than the previous approach, and may take exponential time, it allows us to play against $n$ simultaneous opponents using $O(n)$ space!
2.1. Board representations. The actual space used per game is primarily the representation of a chess position, plus a few bits for bookkeeping. So, representing boards compactly gives us a way to increase the number of simultaneous games we can play for a given storage size.

Mainly, we need to store the pieces and their locations. There are a few other bits, like whose turn it is (1 bit), whether each player can still castle king- and queen-side ( 4 bits ), and whether and where an en passant capture is possible ( 4 bits). ${ }^{3}$

With 64 squares, six pieces for each of the two sides, plus the empty square, a straightforward representation of the board uses $64 \times 4=256$ bits. The Thursd'z Institute considered more compact representations [2]; one nice choice works as follows:

Start with a single 64 -bit mask which indicates, for each square on the board, whether it contains any piece. Note that there can only be up to 32 pieces on the board. To tell what these pieces are, we then follow with 32 four-bit records; these indicate the piece's color and type. ${ }^{5}$ With the 9 bits of extra

[^2]state above, this totals $64+32 \times 4+9=201$ bits. There is some slack in the representation, because there are only 12 actual possibilities for a piece but we can indicate 16 with 4 bits. It is great to abuse this slack to save bits; for example, we can store a new type of rook that is still able to castle (it can only be in the corners and thus also indicates its own color), eliminating the 4 castling bits. We can similarly introduce an en-passantable pawn, saving the 4 bits of en passant state; this piece can only be in ranks 4 or 5 , so it also indicates its color. We can also have a "king whose turn it is" for each side, saving the side-to-move bit. This totals a nice round $64+32 \times 4=192$ bits. ${ }^{6}$ This would allow approximately an 11 billion-game simultaneous exhibition in RAM on my desktop computer.

So now comes the main idea of the paper, which is also spoilered in the very clear paper title. What if we represented boards only as the 64-bit mask telling us what squares are occupied? The encoding is very lossy, of course, but it often contains enough information to deduce the state of the board. For example, can you tell what position this is?


Correct! It is

after 1. Nf3 Nf6 2. e4 Ng 4 3. Bc4 Ne5 4. OO Ng6 5. Kh1 Rg8 6. Nc3 Nh8 7. Qe2 Nc6 8. Rb1 Rb8 9. Nb5 Nb4 10. Rd1 Nd5 11. Qf1 Nb6 12. Qg1 Na8 13. Nbd4 Nb6 14. Rf1 Na8 15. Be2 Ng6 16. Bd1 Nh8 17. Nb3 Ng6 18. Ne1 Nh8 19. Na 1

## 3. Color- And PIECE-BLIND CHESS

So this is kind of like blindfold chess, but for computers! Instead of being blind to the board, and only relying on our memory (trivial for computers), we'll only be able to see where

[^3]pieces are positioned on the board, but not what type or color they are. Of course, we also have to prohibit the computer from simply remembering the board, so the algorithm must be stateless. Specifically, we want a function

```
makemove : uint64 }->\mathrm{ move list
```

that makes a move from a single board position, represented just as 64 bits. This is a single move; the move list represents our preference for the move to make in descending order, and we commit to the first move that is actually legal. It does not work well to insist that the function return a single move, as it will often be the case that the board is misinterpreted and a move is in fact illegal; forcing forfeit ${ }^{7}$ would mean that almost all games end in forfeit, which is boring. On the other hand, allowing the function to try again upon learning a move is illegal would allow it to interactively "interrogate" the board state somewhat. ${ }^{8}$ This seems counter to the spirit of color- and piece-blind chess, so we instead require the function to rank all moves ahead of time.

## 4. Unblinding the board

I went about this by building a function that "unblinds" a board; it has type

$$
\text { unblind : uint64 } \rightarrow \text { position }
$$

This function is natural for machine learning. It is easy to generate copious training data from actual games by simply blinding positions into their corresponding 64-bit numbers; I just randomly sampled 100 million positions from Feburary 2017 on lichess.org.

I repurposed the custom neural network code from my seminal paper Red i removal with artificial retinal networks [3] after discovering that artificial retinal networks are actually isomorphic to neural networks. The main advantage of this code is that it allows for sparse networks, but the real reason to use it is that I would rather spend dozens of hours debugging my own code, pay a larger electric bill, and get worse results in the end, than to spend a short while trying to get someone else's probably-very-good neural network training package to compile.

The structure of the network is as follows. The input layer is 64 nodes, one for each of the 64 bits, with each node set to either $1.0 f$ or $0.0 f$. Three hidden layers of size 1024, 12288, and 567 do the neural magic. The output layer is 837 nodes; the bulk of which is a "one-hot" representation of the predictions for the 64 squares, each with 13 possible contents (black or white, six pieces, or empty). This is $64 \times 13=832$ nodes. Then four nodes to predict the four castling bits, and one to predict the current side to move. This model does not predict the possibility for en passant capture, nor move counters or position repetition. This will not be its main source of disadvantage!

I trained the network in two phases, first starting with a densely connected one (model size 160 MB ), and then after I get fed up with how slow training was, a "vacuumed" version of the network where I removed edges with low absolute weights

[^4](model size 5 MB ) to continue training. Removing edges based on an absolute weight threshold is very unprincipled (since a downstream edge can magnify a contribution arbitrarily) but I did it anyway.

Everything was trained on a single GPU, though a fairly decent one in 2018, the EVGA GeForce GTX 1080 "FTW" (really), using OpenCL. Biases were initialized to 0 and weights with a Gaussian of mean 0 and standard deviation 0.025 . In the first phase, there were 64 examples per round, and after vacuuming, 2048. The round learning rate $\alpha_{r}$ started at 0.1 and descended linearly to 0.002 over 500,000 rounds; and the learning rate when updating weights (for each example) $\alpha$ is $\alpha_{r}$ /examples_per_round. In no way am I recommending these parameters. Fiddling with the parameters to make it do its black magic or alternately carom off to a sea of NaNs or zeroes is for sure the worst part of neural networks. Indeed, I initially started with the classical sigmoid transfer function, but "upgraded" to the "leaky rectified linear" function

$$
(p<0) ? p \times 0.01: p
$$

after getting fed up with sigmoid weights caroming off (see "vanishing gradient problem" and/or "exploding gradient problem"). The final model was trained over 339,885 rounds on 223 million examples. It did not appear to show signs of improving for several days before I terminated it.
4.1. Statistical evaluation. The unblinding component can be evaluated on its own, by running it on examples sampled independently of the training set. The model outputs a score for each possible contents of each square; we simply discretize to the highest-scoring one (same too for the predicted castling state and turn). Over 50,000 examples, these were the results:

9,584 predicted boards were exactly correct ( $19.17 \%$ ). There were a total of 161,166 piece mistakes, which is an average of 3.22 per board. This is wherever we predict a square's contents incorrectly. There were only 1630 castling status mistakes, an average of 0.03 per board (there can be up to four mistakes per board). This is probably because when the king and rook are in their starting squares, castling is almost always still allowed. In 19,014 boards, we mispredicted whose move it is $(38 \%)$. This appears to be the most difficult thing to predict, which is not surprising. ${ }^{9}$
4.2. Subjective evaluation. The unblinder must make mistakes since the 64-bit representation is ambiguous. Subjectively, the unblinder makes reasonable mistakes. It is excellent at common openings, usually getting these exactly correct. On the other hand, it is fairly bad at sparse endgames, where it is difficult to tell a pawn from a rook from a king. It is terrible at unlikely positions that can be confused for likely ones. If you are playing against it and know how it works, it is easy to trick it by doing something like capturing one of its starting-position pawns with your queen; nobody does this in real games (because the queen can be immediately recaptured), so the square

[^5]is predicted as a pawn and the queen "disappears" to the unblinder (Figure 1). Having an invisible queen in your camp is of course very dangerous. Resolving ambiguities in favor of more likely positions is the right thing for the model to do, so this is just an inherent flaw with the decomposition of the problem. There are some ways we can account for this (Section 5.2).

(a)


Figure 1. (a) Position after 1. e4 e5 2. Qg4 d5 3. Qxg7; note the white queen strangely on g7. (b) The bitmask for this position and the unblinder's prediction. The queen "disappears" after Qxg7, because unblinding predicts it to be one of black's own pawns-far more likely in that square.

A few things are distinctly disappointing about its performance. Even outside of "likely" positions, it usually predicts that pieces on black's side of the board are black, and vice versa (Figure 2). This makes sense, but suggests serious limitation on using the prediction to play chess. Less forgivably, it sometimes predicts more than one king per side (or zero), which is always wrong. Actually, an early version had this problem in spades, frequently predicting two or three kings. Upon debugging, I had simply made the noob mistake of printing both King and Knight with the letter "K." Ha! It often predicts the "wrong" number of bishops (etc.), or places them on the same color. This is technically possible through promotion, but usually a bad guess, since promotion is relatively rare, and moreso promotion to a piece other than a queen. An approach that might handle this better (but may have different downsides) would be instead to predict the "fates" of the 32 initial pieces [6]. The fate of a pawn includes what if any piece it has promoted to, but this is not necessary for the other pieces. This would require that the model only predict a single location for each king, among other things. However, this would require a much larger output layer ( 32 pieces can move to 64 squares, plus promotion) and it is not always clear how to interpret its value into a position (for example, if two pieces are predicted strongly to be on the same square).

## 5. Playing Blind

Once we have predicted a board state, we can play with it. The simplest way to do this is to use a strong chess engine to


Figure 2. What the model predicts (in single-king mode; Section 5.1) for OxFFFFFFFFFFFFFFFF, the board with all bits set. This is an impossible position, but it gives some idea of the model's biases for each square. Notably, most of the pieces on each half of the board have a single color. This makes sense, but also suggests substantial limitation. When single-king mode is off, the bottom right king is predicted as a white rook.
pick a move for the position. Here, I use Stockfish with a node budget of 1 million nodes, which takes about 1 second of CPU time per move on my computer. There are some complications:

- Frequently, the unblinded board will not match the true position, and Stockfish will choose a move that is illegal. So, as discussed before, we actually return a prioritized list of moves. For this first experiment, we just return the move Stockfish recommends, followed by all other moves ordered randomly.
- Stockfish is a very strong engine, and in my opinion the code is generally good, but it is very sensitive to bad FEN (the notation used to give it a position to evaluate) strings. Given a bad string, like one that says castling is possible when the king isn't in its home square, often crashes the engine. So we need to make sure to actually pass valid positions. I accomplish this by making the following modifications to the predicted board:
- If a castling bit is set, but castling is not possible due to the position of the king or rook, clear the bit.
- Set the side-to-move to be the correct actual value. This uses the unblinded state, so is superficially cheating. But note that if we get the side wrong, then Stockfish's move will always be illegal: Moves are specified as a source and destination square, ${ }^{10}$ and so the source square of Stockfish's move would always be a piece of the wrong piece's color. So this is equivalent to (but twice as efficient as) running stockfish twice, one for each side, and prioritizing the predicted side's move first.
This won't fix all positions, for example, if the white and black king are adjacent to one another in mutual check. If an irreparable problem is detected, then I just return a uniformly random move list.

[^6]It is easy to beat this chess engine, by tricking it as in Figure 1, although this involves unnatural moves, so it may only apply if you know how it works. Measuring how well it plays in an absolute sense is a subject of some interest, so I wrote a separate paper about that [5]. This algorithm, called blind_yolo, had an Elo World score of $489 \pm 2$. It beats a purely random player with a score of 101 wins, 27 losses, and 389 draws. Making moves purely at random is one of the few fair comparisons, since the random strategy also works with color- and piece-blind chess.
5.1. We three kings. When evaluating the first version I found that it was predicting a disappointingly high number of illegal positions in practice, which was causing us to fall back on making random moves, which is mostly boring. The second version reduces the rate of illegal positions due to too many or too few kings [4].

The model predicts a score for each type of piece in each square, and we do not have to necessarily interpret it by always taking the highest-scoring piece. This version first finds the two squares with the highest scores for the white king, and same for the black king. We take two in case the same square is predicted for both. Then this square gets one of the kings (whichever has higher score) and the other king goes in the highest-scoring unclaimed square. The rest of the squares get the highest-scoring prediction as before, but we never predict kings for them.

This change just affects the unblinding process, so we can directly evaluate its accuracy. It gets $19.28 \%$ of positions exactly correct (slightly better), with an average of 3.26 piece mistakes per position (slightly worse). This is expected; we exchange local mistakes (each was trained independently to minimize its local error) for global correctness (which is not taken into account at all during training).

This version, called blind_kings, performs a small amount better than blind_yolo ( 63 wins, 45 losses, 412 draws). It had an Elo World score of $502 \pm 3$.
5.2. Spy check. Say blind_kings is playing as white; it remains easy to fool it by moving black pieces into white's camp, since they are usually then predicted to be white pieces. We can defend against this somewhat. Since it is illegal to capture one's own piece, there is little risk in trying; if it is indeed our own piece then the move will be rejected, and if it is not our piece, then capturing is good for two reasons: One, we capture a piece, and two, we avoid having Stockfish make a move in this incorrectly predicted board. (Of course there are many reasons why eagerly capturing a piece can be a bad idea, but at this level of play, an edge in material is likely worth it.)

There is one subtlety here. Above we argued that it was safe to use the actual side-to-move instead of the predicted one; but here it would not be equivalent to do so. Instead, we first prioritize all apparent spy-check moves where the predicted source piece matches the predicted side-to-move, then we try the opposite. (Ties are broken by preferring to capture with a lower-value predicted piece, and then randomly.) Due to this, there is some additional chance that we end up making an especially dumb move because we both mispredicted the side-to-move and the identity of some pieces.

This version, blind_spycheck, works significantly better than blind_kings. It has an Elo World score of $547 \pm 1$,
somewhere between a 93.75-96.875\% dilution of stockfish1m (the third best non-engine player).
5.3. Future work. The predicted board often expresses uncertainty about some squares, which could be thought of as probabilities. A principled improvement would be to try to find moves that are good in expectation, that is, integrated over all possible boards in proportion to their predicted probability. A good approximation might be had by sampling a bunch of boards according to the predicted distribution, and then using Stockfish to score the top $k$ moves for each; we can then order moves by their expected score. Unfortunately, it is not easy to efficiently get Stockfish to generate scored moves for $k \neq 1$. Even with $k=1$, this approach would be slow, taking about a second for each (distinct) sampled board. So I did not try it, at least not before submitting this paper.
5.3.1. No, u r a lnetwork. I initially considered trying to solve this whole problem with neural networks. The current best known engine in the world (AlphaZero) at least uses a neural network. The biggest advantage would be that it would naturally be able to consider multiple moves under uncertainty about the board state, as just discussed, without any particular extra logic. My plan was to make multiple different components that could be evaluated separately, starting with the unblinder described, followed by a unit that predicts legal moves, and then a unit that takes these two (and also the 64bit blinded mask if it likes) and scores each move. Predicting a legal move is also a natural function for machine learning; a move can be given just as a source and destination square. ${ }^{11}$ Many pairs of squares are always impossible (e.g. no piece can ever move from A1 to B8); so there are only 1792 potential moves to predict. However, training a reasonable unblinder took longer than I expected, and the legal move predictor never really worked that well (it has a harder job), so I just settled for basing it off the single unblinder unit. Can you do better?

## 6. Conclusion

I would like to thank the little people (pawns) and the author- and content-blind anonymous referrees.

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[^0]:    Date: 1 April 2019.
    Copyright © 2019 the Regents of the Wikiplia Foundation. Appears in The Journal Of LaTeX Class Files with the insufficient material of the Association for Computational Heresy; IEEEEEE! press, Verlag-Verlag volume no. 0x40-2A. 1 tempo.
    ${ }^{1}$ JCVD does not play chess on camera, but it is implied that he is also holding a simultaneous exhibition between rounds in a different room of the underground Hong Kong illegal karate complex.

[^1]:    ${ }^{2}$ There are only 1792 pairs of squares between which pieces can ever move (Section 5.3.1), so $11+2$ bits suffices, with some added complexity.

[^2]:    ${ }^{3}$ Technically, we need to store a lot of additional information with the board in order to completely implement the rules of chess.[1] The trickiest of these involve the rules for draw by repetition, which make reference to the history of the game (See Footnote 1 in Survival in chessland [6]) and seem to require storing all previous board states. Fortunately, if we are being this picky, then we also know that the length of a chess game is bounded by a constant: Rule 9.6.2 ends the game in a draw if both players have made 75 moves without a pawn move or capture, ${ }^{4}$ so it suffices to store the $75 \times 2$ most recent (half-)moves. This sucks so most people don't do it (for example, FEN notation only gives the number of such moves, and so cannot implement the draw by repetition). On the other hand, if we insist, then this may give us a simpler route to a constant-space exhibition, since the $B \xrightarrow{m} B^{\prime}$ relation is probably reversible with such information.
    ${ }^{4}$ These are two types of moves that make it impossible to formally repeat a position that preceded them. Castling also has this property, but doesn't count because it is a "secret move."
    ${ }^{5}$ Since only 32 of the 64 bits can be set, you could do slightly better by representing $\binom{64}{32}$ in $\sim 61$ bits. When fewer than 32 squares are occupied, we can use a record containing e.g. a third king (otherwise impossible) to indicate that we should ignore the remaining bits. However, this gets vastly more complicated for only 3 bits of savings.

[^3]:    ${ }^{6}$ Since this is SIGBOVIK, I am freed from the burden of comparing related work. I did however read the rather bad Wikipedia article on the topic [7] which describes a Huffman-based encoding that uses a "maximum of 204 bits, and often much less." This also includes a 7 -bit 50-move counter (but you really need to implement a 75 -move counter; 50 moves only allow a player to claim a draw) so should be compared as 197 bits. But the article also contains many bugs, like the misconception that there can only be four total rooks (pawns are allowed to promote to rook). So the approach described here is both more efficient and more correct.

[^4]:    ${ }^{7}$ FIDE rules state that the second attempt at an illegal move results in forfeit (7.5.5).
    ${ }^{8}$ Similar to Kriegspiel [9], although in that game at least one's own pieces are known!

[^5]:    ${ }^{9}$ Prior to "vacuuming", the 160 MB network actually performed slightly better than the final 5 MB network, with $21.20 \%$ of boards exactly correct, and an average of 3.12 mistakes per board. This suggests that the model may only be doing a limited amount of generalization, instead mostly memorizing board positions. Representing the 223 million examples seen exactly (using our best board representation described in Section 2.1) would take 42.8 GB , so at 5 MB at least the data is represented compactly, if also rather lossily.

[^6]:    ${ }^{10}$ Plus promotion piece. Castling is represented as a two-square move of the king.

[^7]:    ${ }^{11}$ There are also four choices for promotion when moving a pawn into the last rank. It is always the case that if any promotion is legal, all choices are legal, so this does not need to be encoded in this phase. Also, at this level of play, always promoting to queen is a very safe simplification.

